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The algebraic generator coordinate method for locally compact Lie groups

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Abstract. A generalization of the algebraic generator coordinate method to the case of locally compact Lie groups is obtained. This generalization allows the spaces of collective states generated by the non-compact Lie groups to be constructed. A scheme for the derivation of the coherent states in the collective spaces is shown.

1. Introduction

The algebraic generalization of the generator coordinate method (AGCM) has been proposed in [1]. The $L^2(G)$ convolution algebra with involution, where G was a *compact group of motions*, played the main role in this approach. The compactness of the group of motion G was the crucial point for some proofs in the method described in [1]. In the present paper we extend this approach to the case of locally compact groups which are in many applications more interesting than compact ones. The extension is not straightforward because of the different structure of the algebra $L^1(G)$ that has to be taken into account, compared to the more familiar algebra $L^2(G)$. However, most of the useful relations known from the compact case can also be proved for non-compact groups. This is shown in section 2 of the paper. Section 3 contains a group theoretical application of the AGCM approach in construction of the coherent states for a class of representations of compact and non-compact groups. For convenience we will now sketch the main ideas of the standard GCM formalism.

The GCM, originally described by Hill, Wheeler and Griffin [2] and in its up to date version presented in [3] (for a review of the literature see [4]), is a quantum mechanical tool that enables us to construct the spaces of states having some required properties. The main object in this method is a family of states labelled by a set of parameters called the generator coordinates. The family of states is named the generator function. In the traditional variational approach to the GCM the generator function $|q\rangle = |q_1, q_2, \ldots, q_m\rangle$ defines a very general ansatz for a trial function in the following form:

$$|\Psi\rangle = \int \mathrm{d}q\, f(q)|q\rangle. \tag{1}$$

The integral is an m-dimensional integral over the space of q-parameters. The function $f(q)$ is a wavelike amplitude and can be determined from the variational principle

$$\delta\langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle = 0 \tag{2}$$

where H is the Hamiltonian of the physical system. Equation (2) leads to an integral

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equation for the weight function $f(q)$. The lowest eigenvalue of this so-called Griffin–Hill–Wheeler integral equation [2] should give an approximate ground state energy of the system while higher eigenvalues coincide with the excited states. The set of states Ψ generated by (1) forms a subspace $\mathscr{H}_\Psi$ of the complete Hilbert space $\mathscr{H}$ of the wavefunctions of the original physical system. Solving the integral equation for $f(q)$ is tantamount to diagonalizing the complete Hamiltonian of the system in the subspace $\mathscr{H}_\Psi$ of $\mathscr{H}$. The Griffin–Hill–Wheeler method gives exact solutions if exact states Φ belong to the subspace $\mathscr{H}_\Psi$. In other cases the results obtained by the GCM are approximate eigenfunctions and eigenvalues. The accuracy of the method depends on the extent to which the eigenstates of H can be approximated by states in $\mathscr{H}_\Psi$. This in turn depends on the choice of the generator function $|q\rangle$.

The states (1) are pure states in a quantum mechanical sense and it is a very interesting problem to extend the method in such a way to generate the collective spaces from a fixed-density operator. The generalization of the GCM which comprises the mixed states was proposed in [1] and illustrated by a construction of a collective space generated from a density matrix of the system under consideration. The generator coordinates (the collective variables) were established there by means of the appropriate group of motions; a compact Lie group.

However, the compact groups are often inadequate to describe various motions which are responsible for many kinds of excitations in many-particle systems. Also, the compact groups do not reveal continuous spectra for unbounded states. Thus, it will not be amiss to pay more attention to the locally compact groups.

2. AGCM

Let G be a locally compact Lie group and for simplicity unimodular. In this case one cannot use the involutive convolution algebra $L^2(G)$ as in the case of compact groups [1] because now the convolution of two functions belonging to $L^2(G)$ is not necessarily a 'square integrable' function. Instead, we have to introduce the Banach algebra $L^1(G)$ of complex functions on the group G with involution # given by the relation [5]

$$u^{\#}(g) = u^*(g^{-1}) \tag{3}$$

where * indicates the usual complex conjugation. The multiplication law in the $L^1(G)$ algebra is, as usual, established by means of the convolution in the following form:

$$(u \circ v)(g) = \int_G \mathrm{d}g'\, u(g')v(g'^{-1}g) \tag{4}$$

where $\mathrm{d}g$ denotes the Haar measure on G.

On the group G we define the complex function $\langle\rho; \cdot\rangle$ which fulfils the following three conditions:

(i) $\langle\rho; g^{-1}\rangle = \langle\rho; g\rangle^*$. $\qquad (5a)$

(ii) For any finite sequence $\alpha_1, \ldots, \alpha_n$ of complex numbers and any arbitrary sequence $g_1, g_2, \ldots, g_n$ of points on the group manifold. $n = 2, 3, \ldots$, the following relation is fulfilled:

$$\sum \alpha_i^* \alpha_j \langle\rho; g_i^{-1} g_j\rangle \geqslant 0. \tag{5b}$$

(iii) The function defined by the integral

$$\int_G \mathrm{d}g' \langle\rho; g^{-1}g'\rangle u(g') \tag{5c}$$

belongs to the algebra $L^1(G)$.

Using the invariance of the Haar measure one can see by direct calculations that for arbitrary $\langle\rho;\cdot\rangle \in L^1(G)$ the condition (iii) is always fulfilled.

We use the function $\langle\rho;\cdot\rangle$ to define the positive linear functional on the algebra $L^1(G)$. We denote the functional by the same symbol $\langle\rho;\cdot\rangle$ because it is easy to distinguish between the function and the functional specifying its argument. The functional $\langle\rho;\cdot\rangle$ is defined as follows:

$$\langle\rho; u\rangle = \int_G \mathrm{d}g\, u(g)\langle\rho; g\rangle. \tag{6}$$

To have a continuous functional one needs to assume the additional condition for the function (5), namely the function $\langle\rho;\cdot\rangle$ has to belong to the space of essentially bounded functions usually denoted by $L^\infty(G)$ [6].

It will now be worthwhile to list some properties of (6) which will be useful in further applications. It turns out that:

(i) The only functionals given by (6) are linear and continuous in $L^1(G)$. The functional (6) has the norm equal to $\|\langle\rho; g\rangle\|_\infty$. These statements relate to theorem IV.3.5 in [7].

(ii) Straightforward calculations reveal another feature of (6), namely

$$\langle\rho; u^\#\rangle = \int_G \mathrm{d}g\, u^\#(g)\langle\rho; g\rangle = \int_G \mathrm{d}g\, u^*(g)\langle\rho; g^{-1}\rangle = \langle\rho; u\rangle^*.$$

This relation can be derived by means of (3), (5a), and of the invariance property of the Haar measure on G.

(iii) On the basis of (5b) $\langle\rho; u^\# \circ u\rangle$ is greater or equal to zero for all u from $L^1(G)$.

The next step in our procedure is to define the overlap operator, known also in the GCM, as follows:

$$(\mathcal{N}u)(g) = \int_G \mathrm{d}g'\langle\rho; g^{-1}g'\rangle u(g'). \tag{7}$$

For the integral kernel $\langle\rho;\cdot\rangle$ belonging to $L^1(G)$ the operator $\mathcal{N}$ is a well defined continuous operator in the algebra $L^1(G)$ because the integral in (7) can be rewritten in the form of the convolution of two functions giving a function from $L^1(G)$, i.e.

$$(\mathcal{N}u)(g) = \int_G \mathrm{d}g'\, u(g')\langle\rho; g'^{-1}g\rangle^*.$$

Using definition (7) one can write the functional of the convolution of two functions in the form of a single integral of a local function:

$$\langle\rho; \alpha^\# \circ \beta\rangle = \int_G \mathrm{d}g\, \alpha^*(g)(\mathcal{N}\beta)(g). \tag{8}$$

Because the functional (6) is positively defined, i.e. $\langle\rho; u^\# \circ u\rangle \geq 0$. The overlap operator has an analogous property, i.e.

$$\int_G \mathrm{d}g\, \alpha^*(g)(\mathcal{N}\alpha)(g) \geq 0 \qquad \text{for all } \alpha \in L^1(G). \tag{9}$$

Following the idea of the AGCM described in [1] one can construct the space of states by means of the GNS construction [8]. By this procedure one can obtain the space of states determined by the group of motion G and the functional $\langle\rho;\cdot\rangle$. The functional (6) allows the introduction of the scalar product in the obtained linear space, which can be turned into the Hilbert space after the standard completion. As a result one obtains the Hilbert space of states in which the action of the algebra $L^1(G)$ and the group G itself is defined in a very natural way. Below, we also show the connection between this construction and the standard GCM which explains the physical meaning of the construction in a more intuitive way.

By the idea of the GNS construction, (8) could be used as a scalar product in the space $L^1(G)$, but a non-zero element from $L^1(G)$ may have a zero norm determined by this scalar product. This problem is straightforwardly related to the zero eigenvalues problem of the overlap operator in the GCM approach [4, section 10.2.2].

To put these pathological behaviours aside one can define the left-ideal in the algebra $L^1(G)$ that comprises all the pathological elements:

$$\mathcal{R}_\rho = \{u \in L^1(G); \langle\rho; \alpha^{\#} \circ u\rangle = 0 \text{ for all } \alpha \in L^1(G)\}. \tag{10}$$

These null elements can be interpreted as the states indistinguishable by the group of motion G of the physical system under consideration. Obviously, during the evolution of the system the functional (6), in general, can also evolve and the left-ideal (10) can change with time, giving another set of indistinguishable states, i.e. the space of states can vary with time or other external parameters like strength of fields, temperature, static deformations, etc.

A possible practical method to find all the elements belonging to $\mathcal{R}_\rho$ is by use of the overlap operator $\mathcal{N}$, (7). Using the standard proof one can show the generalized Schwartz–Cauchy inequality within the algebra $L^1(G)$:

$$|\langle\rho; u^{\#} \circ v\rangle|^2 \leqslant \langle\rho; u^{\#} \circ u\rangle\langle\rho; v^{\#} \circ v\rangle. \tag{11}$$

Applying this inequality to the definition of $\mathcal{R}_\rho$ one can see that $u \in \mathcal{R}_\rho$ if and only if $\langle\rho; u^{\#} \circ u\rangle = 0$. At this point it is worthwhile noting that one can obtain a very simple relation directly from the definition of the overlap operator (7):

$$\langle\rho; u\rangle = (\mathcal{N}u)(e) \tag{12}$$

where e denotes a unit element in G. A consequence of this relation is that for u is equal to 0 when it belongs to the left-ideal $\mathcal{R}_\rho\langle\rho; u\rangle$. This statement has to be proved in this indirect way because the algebra $L^1(G)$ does not contain unity.

For a fixed β, (8) determines a continuous functional on $L^1(G)$ and this implies that the left-ideal (10) can be identified with the kernel of the overlap operator, i.e. $\mathcal{R}_\rho = \mathrm{Ker}(\mathcal{N})$, and one needs to solve the following equation:

$$\mathcal{N}u = 0. \tag{13}$$

Having found the left-ideal (10) one can construct a pre-Hilbert space $\mathcal{H}$, which emerges as a quotient of the algebra $L^1(G)$ and the left-ideal $\mathcal{R}_\rho$, i.e. $\mathcal{H} = L^1(G)/\mathcal{R}_\rho$. After the standard completion procedure of $\mathcal{H}$ one obtains the Hilbert space of states denoted by the same symbol $\mathcal{H}$, generated by the functional $\langle\rho; \cdot\rangle$ and the group of motion G. The scalar product in $\mathcal{H}$ is given by the relation

$$(u|v)_{\mathcal{H}} = \langle\rho; u^{\#} \circ v\rangle. \tag{14}$$

We remark that on the left-hand side of (14) one needs to write classes of equivalent elements which are elements of the space $\mathcal{H}$, but, following the usual convention for convenience, one can use their representatives instead.

Expression (14) can be rewritten as the double integral over the group manifold:

$$(u|v)_{\mathscr{K}} = \int_G \mathrm{d}g' \int_G \mathrm{d}g\, u^*(g')\langle\rho; g'^{-1}g\rangle v(g). \tag{15}$$

The obtained scalar product couples the states by the non-local function which is a generalization of a standard overlap function from the GCM approach. This overlap contains some correlations between different values of the generator coordinates represented here by g and g'. At this point it will be meaningful to link the AGCM to the standard GCM.

For this purpose let us consider the case of the function $\langle\rho; \cdot\rangle$ defined by means of the scalar product in a Hilbert space $\mathscr{H}$, e.g. the many-body space of states of the physical system, as follows:

$$\langle\rho; g\rangle = \langle -|T(g)|-\rangle \tag{16}$$

where $T(g)$ denotes a unitary representation of group G in Hilbert space and $|-\rangle$ is an arbitrary (but in applications having a physical meaning) state vector in $\mathscr{H}$. As a first step we prove the following lemma.

Lemma 1. $u \in \mathscr{R}_\rho$ iff $\|\int_G \mathrm{d}g\, u(g)T(g)|-\rangle\|_{\mathscr{H}} = 0$, i.e. the vector $|u\rangle \equiv \int_G \mathrm{d}g\, u(g)T(g)|-\rangle = 0$ in the many-body Hilbert space $\mathscr{H}$ in which the generator function $|-\rangle$ is determined.

Proof. Suppose $u \in \mathscr{R}_\rho$, then $\mathscr{N}u = 0$. The last condition can be rewritten in the form $\langle -|T^+(g)|u\rangle = 0$ for all $g \in G$. Multiplying both sides of the equations by $u^*(g)$ and integrating over g one obtains that the scalar product $\langle u|u\rangle = 0$. Conversely, from the fact that $|u\rangle = 0$ for every vector $|v\rangle \in \mathscr{H}$ $\langle v|u\rangle = 0$, i.e. also the functional $\langle\rho; v^{\#} \circ u\rangle = 0$ for all $v \in L^1(G)$ and $u \in \mathscr{R}_\rho$. □

From lemma 1 one can directly see that the null space $\mathscr{H}_0$ of the GCM approach consists of vectors of the form

$$\mathscr{H}_0 = \left\{\int_G \mathrm{d}g\, u(g)T(g)|-\rangle : u \in \mathscr{R}_\rho\right\} \tag{17}$$

and the ratio $\mathscr{H}/\mathscr{H}_0$ after completion gives the GCM space $\mathscr{H}_{\mathrm{GCM}}$. Another direct consequence of lemma 1 is that there exists a one-to-one unitary transformation between the spaces $\mathscr{K}$ and $\mathscr{H}_{\mathrm{GCM}}$, defined as

$$\tau: \quad \mathrm{cl}_{\mathscr{K}}(u) \to \mathrm{cl}_{\mathrm{GCM}}\left(\int_G \mathrm{d}g\, u(g)T(g)|-\rangle\right) \tag{18}$$

where $\mathrm{cl}(\cdot)$ denotes vectors in the spaces $\mathscr{K}$ or $\mathscr{H}_{\mathrm{GCM}}$ which comprise some classes of the corresponding equivalent elements from $L^1(G)$ and $\mathscr{H}$. The unitarity of τ follows directly from the forms of the scalar products in $\mathscr{K}$ and $\mathscr{H}$.

In many cases it is important to know how the group G acts on the space $\mathscr{K}$. Following [1] we define the action of G by the left shift operator as follows:

$$\mathscr{L}(g')u(g) = u(g'^{-1}g) \qquad \text{where } u \in \mathscr{K}. \tag{19}$$

The operators $\mathscr{L}(g)$ furnish a unitary representation of the group G in the space $\mathscr{K}$. On the other hand, calculating the τ-transformation of the vector $\mathscr{L}(g)\,\mathrm{cl}_{\mathscr{K}}(u)$ one obtains the vector $T(g)\tau(\mathrm{cl}_{\mathscr{K}}(u))$ belonging to the GCM space $\mathscr{H}_{\mathrm{GCM}}$, i.e.

$$\tau\mathscr{L}(g) = T(g)\tau. \tag{20}$$

Equation (20) means that the operator τ is the intertwining operator for the representations $\mathscr{L}(g)$ and $T(g)$.

In this way we have shown that the formalism presented here is the extension of the AGCM approach to locally compact groups. It can also be applied to the compact case, sometimes enlarging in this way the set of available vectors, in comparison with the $L^2(G)$ algebra, in the resultant construction of the state space. The choice of the algebra $L^p(G)$, $p=1$ or 2, for the compact case is dependent on our demands concerning the possible analytical behaviour of state vectors of a physical system.

For simplicity, we have only considered here unimodular locally compact groups. However, the extension to non-unimodular locally compact groups is straightforward and leads to modifications of the formulae by insertion of the modular functions and the appropriate Radon–Nikodym derivatives.

3. Coherent states in the space $\mathscr{K}$

Since their introduction, a variety of coherent states based on different groups has been constructed and widely applied in physics [9]. A general prescription for the construction of coherent states exists only for semisimple Lie groups [10]. However, many Lie groups which are of great interest in physics, e.g. the Poincaré group or Euclidean group, do not belong to this 'well behaved' category. The main difficulty in constructing a system of coherent states for these groups is the fact that, considering for example the Euclidean groups in two or more dimensions, the faithful, unitary, irreducible and square integrable representation does not exist. This feature does not allow the fundamental resolution of unity that characterizes coherent states.

Our aim in this section is to show under which conditions the unitary representation $\mathscr{L}$ acting in the carrier space $\mathscr{K}$ defined by (19) is square integrable and allows resolution of unity. We follow an excellent idea of Isham and Klauder presented in [11]. The solution suggested by these authors is to use the unitary, faithful, but not necessarily irreducible, representations. In our case the representation $\mathscr{L}$ is unitary; however, as will be shown below, irreducibility and other properties of the representation $\mathscr{L}$ are dependent on the form of the functional $\langle\rho;\cdot\rangle$.

First we show under which conditions the unitary representation $\mathscr{L}$ is faithful, i.e. the equality $\mathscr{L}(g_1)=\mathscr{L}(g_2)$ implies the equality $g_1=g_2$. Let us define the subset of group elements

$$F_\rho=\{g:\langle\rho;g\rangle=\langle\rho;(g_1^{-1}g'g_1)g\rangle \text{ for all } g_1, g \text{ belonging to } G\} \qquad (21)$$

which will be helpful in further considerations. Let us suppose now that the representation $\mathscr{L}$ is not faithful, i.e. there exists a pair of group elements $g_1\neq g_2$ that for every vector $u\in\mathscr{K}$ $\mathscr{L}(g_1)u=\mathscr{L}(g_2)u$. This condition can be rewritten in the following form:

$$\mathscr{L}(g_0)u=u \qquad \text{where } g_0=g_1^{-1}g_2. \qquad (22)$$

The vector u can also be represented by an element of the algebra $L^1(G)$, which we denote by the same symbol u. Within the algebra, (22) is equivalent to the statement that $\{\mathscr{L}(g_0)u-u\}$ belongs to the left-ideal (null vector of the space $\mathscr{K}$) $\mathscr{R}_\rho$, and instead of (22) one can write the following condition:

$$\langle\rho;v^{\#}\circ(\mathscr{L}(g_0)u-u)\rangle=0 \qquad \text{for all } u,v\in L^1(G). \qquad (23)$$

In a more explicit form, (23) can be expressed as

$$\int_G \mathrm{d}g_1\int_G \mathrm{d}g_2\, v^*(g_1)\langle\rho;g_1^{-1}g_2\rangle(u(g_0^{-1}g_2)-u(g_2))=0 \qquad (24)$$

for every u and v belonging to $L^1(G)$. Because the function v is arbitrary, the following equation is fulfilled for every g_1:

$$\int_G dg_2\langle\rho; g_1^{-1}g_2\rangle(u(g_0^{-1}g_2)-u(g_2))=0. \tag{25}$$

Use of the invariance of the Haar measure over G allows (25) to be rewritten as

$$\int_G dg_2(\langle\rho; g_1^{-1}g_0g_2\rangle-\langle\rho; g_1^{-1}g_2\rangle)u(g_2)=0 \tag{26}$$

for every $u\in L^1(G)$ and $g_1\in G$. Equation (26) implies the equality

$$\langle\rho; g\rangle=\langle\rho; g_1^{-1}g_0g_1g\rangle \tag{27}$$

for all g and g_1 belonging to G. More precisely, all the above equations are fulfilled almost everywhere, i.e. everywhere except, eventually, a set of the measure zero. These considerations lead to the following statement: if the set (21) $F_\rho=\{e\}$, where e denotes unity in the group G, then the representation $\mathscr{L}$ is faithful.

Assume now that the representation $\mathscr{L}$ of G in $\mathscr{H}$ is unitary and faithful. The representation $\mathscr{L}$ is called 'square integrable' if

$$\int_G dg|(u|\mathscr{L}(g)u)_{\mathscr{H}}|^2<\infty \tag{28}$$

for every $u\in L^1(G)$. One can consider the following sequence of inequalities:

$$\begin{aligned}0\leq\int_G dg|(u|\mathscr{L}(g)v)_{\mathscr{H}}|^2&=\int_G dg(u|\mathscr{L}(g)v)_{\mathscr{H}}(u|\mathscr{L}(g)v)^*_{\mathscr{H}}\\
&=\int_G dg_1\int_G dg_2\int_G dg_3\int_G dg_4\, u^*(g_1)v(g_2)u(g_3)v^*(g_4)\\
&\quad\times\int_G dg\langle\rho; g\rangle\langle\rho; g_3^{-1}g_1gg_2^{-1}g_4\rangle^*\\
&\leq\int_G dg_1\int_G dg_2\int_G dg_3\int_G dg_4|u^*(g_1)v(g_2)u(g_3)v^*(g_4)|\\
&\quad\times\int_G dg\langle\rho; g\rangle\langle\rho; g_3^{-1}g_1gg_2^{-1}g_4\rangle^*\\
&\leq\|\langle\rho;\cdot\rangle\|_\infty\|\langle\rho;\cdot\rangle\|_1\|u\|_1^2\|v\|_1^2\end{aligned} \tag{29}$$

where $\|x\|_p$ denotes the norm in $L^p(G)$ space. The last expression is finite if the function $\langle\rho;\cdot\rangle$ belongs simultaneously to both the spaces $L^1(G)$ and $L^\infty(G)$, and in this case the representation $\mathscr{L}$ is square integrable.

Now, if the representation $\mathscr{L}$ in the space $\mathscr{H}$ is unitary, faithful and square integrable then following [11] one can define the mapping

$$\chi:\quad \mathscr{H}\ni\Psi\rightarrowtail\Psi_\eta(g)=(\mathscr{L}(g)\eta|\Psi)_{\mathscr{H}}\in L_{\mathscr{H}}(G)\subset L^2(G) \tag{30}$$

where η is a fiducial vector [11]. If the mapping χ is one-to-one then one can write the fundamental resolution of unity as follows:

$$\int_G dg\,\Psi_\eta(g)^*\Phi_\chi(g)=\int_G dg(\Psi|\mathscr{L}(g)\eta)_{\mathscr{H}}(\mathscr{L}(g)\eta|\Phi)_{\mathscr{H}}=c(\Psi|\Phi)_{\mathscr{H}} \tag{31}$$

where c is a constant dependent on the normalization of the fiducial vector η. Equation (31) can be expressed in the operator form characteristic for coherent states as

$$c^{-1}\int_G \mathrm{d}g|\mathscr{L}(g)\eta)_{\mathscr{K}\mathscr{K}}(\mathscr{L}(g)\eta| = 1. \tag{32}$$

For the special case of the irreducible representation $\mathscr{L}$ the mapping χ is always an injection. This statement is a consequence of the very well known generalized Schur's lemma; see for example [12].

4. Conclusions

In this paper a natural extension of the AGCM on to locally compact groups of motion has been proposed. In this generalization instead of the *-algebra $L^2(G)$ the *-algebra $L^1(G)$, which is well defined for both types of Lie groups, i.e. for compact and locally compact groups, is applied. The generalization of the overlap operator which allows for searching of the null space generated by the metastate is obtained. A straightforward link to the standard GCM, similarly to the compact case, is also shown. As an application a general construction of the coherent states, widely exploited in physics, for group representations obtained as a result of the AGCM is derived.

Taking into account the results obtained in [1] and this generalization one can conclude that this method has the advantages of the standard GCM and the power of group theoretical approaches.

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